Musical scales are based on Fibonacci numbers

The Fibonacci series appears in the foundation of aspects of art, beauty and life. Even music has a foundation in the series as: There are 13 notes in the span of any note through its octave. A scale is composed of 8 notes, of which the 5th and 3rd notes create the basic foundation of all chords, and are based on whole tone which is 2 steps from the root tone that is the 1st note of the scale.

The piano keyboard scale of C to C above the 13th keys has 8 white keys and 5 black keys, split into groups of 3 and 2.
In a scale the dominant note is the 5th note of the major scale, which is also the 8th note of all 13 notes that comprise the octave. This provides an added instance of Fibonacci numbers in key musical relationships. Interestingly, 8/13 is 0.61538, which approximates phi. What's more the typical three song chord in the key of A is made up of A, its Fibonacci and phi partner E and D, to which A bears the same relationship as E does to A. this is analogous to the 'A is to B as B is to C' basis of the golden section, or in this case 'D is to A as A is to E

How is the golden ratio used in music?

Golden ratio is a 'ratio' and in music we can refer to this ratio to spatial dimensions as well as to temporal dimensions. In the art of painting and generally in all plastic and figurative arts the implementation of a dimension is relatively simple whereas in the art of music, the implementation of a dimension ratio becomes more and more complicated.

In music the term interval describes the relationship between the pitches of two notes. Those intervals may be described as **harmonic (vertical)** if the two notes sound simultaneously, or **melodic (linear)** if the notes sound successively. Intervals may be labeled according to the ratio of frequencies of the two pitches and you can at first implement and apply the above shown dimension ratio to those frequencies: you will have a first example of the golden ratio in music. Furthermore in music we can also consider the intervals of time that is the time intervals between a note and another one.

The Golden Ratio appears in the relationship of the intervals or distance between the notes. Each of these intervals or note pairs creates either a tonic (consonant) sound or a dissonant sound, in which the listener desires to hear it followed by a tonic sound to "resolve" the tension created by its unstable quality.

The octave has a similar consonant quality that could be represented visually by two squares of equal size. A correlation could be made between the consonant properties of the interval of an octave to the first two squares in the golden rectangle or the first two numbers in the Fibonacci sequence which are represented as 1,1. From there the relationship reflects a ratio of 8:5 in the interval of a Major 6th (an approximate Golden Ratio of 1.6), in the first and sixth notes of a diatonic (Major) scale. The ratio of this interval is related by the rhythmic beats that are created by the respective frequencies of the sound waves and interpreted as sound in the human ear. Suffice it to say, that the interval of a Major 6th is supposed to be the most aesthetically pleasing since it contains the golden ratio.

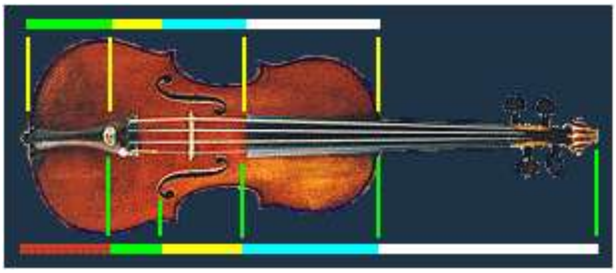
The **interesting** part of this, according to H.E. Huntley, author of the ***Divine Proportion***, is that it is a relationship of the consonance and dissonance of the rhythmic "beats" that occur in the sound waves of the resonant frequencies between notes in the diatonic scale. Huntley goes on to explain that the reason that we prefer visual aspects of a Golden Rectangle over a perfect square is measured in the amount of time it takes for the human eye to travel within its borders. This period of time is in same proportion (Phi) to the beats that exist in specific musical intervals. Unison (two notes of the same frequency being played simultaneously) is said to be the most consonant, having a rhythmic quality that is similar to the time interval that is perceived by the eye when viewing a perfect square.

The difficulty of using the Golden Ratio

In music the term interval describes the relationship between the pitches of two notes. Intervals may be labeled according to the ratio of frequencies of the two pitches and you can at first implement and apply the above shown dimension ratio to those frequencies: you will have a first example of the golden ratio in music. But, if this implementation can be relatively applicable to harmonic intervals, it can be a real destruction of the art for melodic intervals. Furthermore in music we can also consider the intervals of time that is the time intervals between a note and another one. Now, the implementation of dimension ratio becomes artistically restrictive in a dramatic way. So, you can find the Golden Ratio in music as in other arts but the difficulty is to compose using the Golden Ratio as a starting point without destroying the art nature.

However, regarding the listener, the degree to which the application of the golden ratio in music is salient, whether consciously or unconsciously, as well as the overall musical effect of its implementation, if any, is unknown.

Musical instruments are often based on phi



Fibonacci and phi are used in the design of violins and even in the design of high quality speaker wire.

Composers that used the golden ratio and how they used it

James Tenney reconceived his piece *For Ann (rising)*, which consists of up to twelve computer-generated upwardly glissandoing tones (see Shepard tone), as having each tone start so it is the golden ratio (in between an equal tempered minor and major sixth) below the previous tone, so that the combination tones produced by all consecutive tones are a lower or higher pitch already, or soon to be, produced.

Ernő Lendvai analyzes Béla Bartók's works as being based on two opposing systems, that of the golden ratio and the acoustic scale, though other music scholars reject that analysis. In Bartok's *Music for Strings, Percussion and Celesta* the xylophone progression occurs at the intervals

1:2:3:5:8:5:3:2:1. French composer Erik Satie used the golden ratio in several of his pieces, including *Sonneries de la Rose+Croix*.

The golden ratio is also apparent in the organization of the sections in the music of Debussy's *Reflets dans l'eau (Reflections in Water)*, from *Images* (1st series, 1905), in which "the sequence of keys is marked out by the intervals 34, 21, 13 and 8, and the main climax sits at the phi position."

This Binary Universe, an experimental album by Brian Transeau (popularly known as the electronic artist BT), includes a track titled *1.618* in homage to the golden ratio. The track features musical versions of the ratio and the accompanying video displays various animated versions of the golden mean.

The musicologist Roy Howat has observed that the formal boundaries of *La Mer* correspond exactly to the golden section. Trezise finds the intrinsic evidence "remarkable," but cautions that no written or reported evidence suggests that Debussy consciously sought such proportions. Also, many works of Chopin, mainly Etudes (studies) and Nocturnes, are formally based on the golden ratio. This results in the biggest climax of both musical expression and technical difficulty after about 2/3 of the piece.

Pearl Drums positions the air vents on its Masters Premium models based on the golden ratio. The company claims that this arrangement improves bass response and has applied for a patent on this innovation.

In the opinion of author Leon Harkleroad, "Some of the most misguided attempts to link music and mathematics have involved Fibonacci numbers and the related golden ratio."

MOZART

J.F. Putz, a mathematician, has measured some of Mozart's works. Mozart's piano sonatas were convenient targets, because in Mozart's time they were customarily divided into two parts: (1) Exposition; and (2) Development and Recapitulation. Sure enough, the first movement of Mozart's Sonata No. 1 in C Major consists of 100 measures that are divided into the customary two parts; 38 in the first, 62 in the second. This ratio 38/62 (0.613) is as close as one can get to 0.618 in a composition of 100 measures. The second movement of this sonata is also divided according to the Golden Section, but the third movement is not. Many other Mozart piano sonatas seem to employ the Golden section, but some deviate considerably. So Putz could not really claim that Mozart consciously used the Golden Section to "improve" his music (Question #1 above), but there are certainly a lot of "coincidences."

DEBUSSY

Given that Debussy's music is apparently so concerned with mood and colour, one may be surprised to discover that, according to Howat, many of his greatest works appear to have been structured around mathematical models even while using an apparent classical structure such as sonata form. Howat suggests that some of Debussy's pieces can be divided into sections that reflect the golden ratio, frequently by using the numbers of the standard Fibonacci sequence. Sometimes these divisions seem to follow the standard divisions of the overall structure. In other pieces they appear to mark out other significant features of the music. The 55 bar-long introduction to 'Dialogue du vent et la mer' in *La Mer*, for example, breaks down into 5 sections of 21, 8, 8, 5 and 13 bars in length. The golden mean point of bar 34 in this structure is signalled by the introduction of the trombones, with the use of the main motif from all three movements used in the central section around that point.

The only evidence that Howat introduces to support his claim appears in changes Debussy made between finished manuscripts and the printed edition, with the changes invariably creating a Golden Mean proportion where previously none existed. Perhaps the starkest example of this comes with *La cathédrale engloutie*. Published editions lack the instruction to play bars 7-12 and 22-83 at twice the speed of the remainder, exactly as Debussy himself did on a piano-roll recording. When analysed with this alteration, the piece follows Golden Section proportions. At the same time, Howat admits that in many of Debussy's works, he has been unable to find evidence of the Golden Section (notably in the late works) and that no extant manuscripts or sketches contain any evidence of calculations related to it

BARTOK

Béla Bartók memorial plaque in Baja, Hungary

Erno Lendvai (1971) analyses Bartók's works as being based on two opposing tonal systems, that of the acoustic scale and the axis system, as well as using the golden section as a structural principle.

Per Nørgård

Nørgård has composed works in all major genres: six operas, two ballets, seven symphonies and other pieces for orchestra, several concertos, choral and vocal works, an enormous number of chamber works, ten string quartets and several solo instrumental works.

In the 1960s, Nørgård began exploring the modernist techniques of central Europe, eventually developing a serial compositional system based on the "infinity series" (Nørgård 1975), which he used in his

Voyage into the Golden Screen, the Second and Third Symphonies, *I Ching*, and other works of the late 1960s and 70s (Mortensen [n.d.]).

Nørgård is also a prolific writer. He has written many articles dealing with music not only from a technical but also a philosophical viewpoint.

Music


Uendelighedsrækken in G major (the first 32 terms)

Nørgård's music often features the use of the **infinity series** for serializing melody, harmony, and rhythm in musical composition. The method takes its name from the endlessly self-similar nature of the resulting musical material, comparable to fractal geometry. Mathematically, the infinity series is an integer sequence (Fibonacci sequence is an integer sequence). The first few terms of its simplest form are 0, 1, −1, 2, 1, 0, −2, 3, … (sequence A004718 in OEIS).

Nørgård discovered the melodic infinity series in 1959 and it proved an inspiration for many of his works during the 1960s. However, it was not until his *Voyage into the Golden Screen* for small ensemble (1968)—which has been identified as the first "properly instrumental piece of spectral composition" (Anderson 2000, 14)—and Symphony No. 2 (1970) that it provided the structure for an entire work (Nørgård 1975, 9). The harmonic and rhythmic infinity series were developed in the early 1970s and the three series were first integrated in Nørgård's Symphony No. 3.

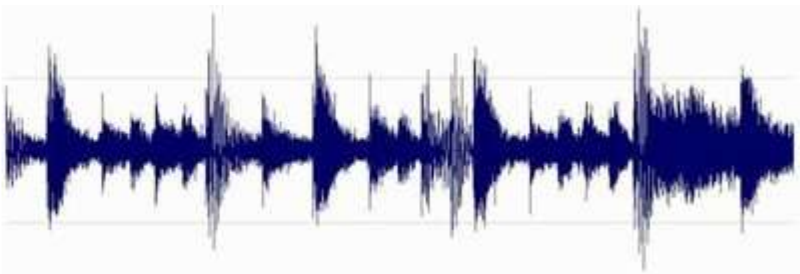
Chopin

Many works of Chopin, mainly Etudes (studies) and Nocturnes, are formally based on the golden ratio. This results in the biggest climax of both musical expression and technical difficulty after about 2/3 of the piece.

Opposite beliefs

Some people believe that all the claims that some Gregorian chants are based on the Golden Ratio, that Mozart used the Golden Ratio in some of his music, and that Bartok used the Golden Ratio in some of his music are without any concrete support. They also say that it is less clear cut whether Debussy used the Golden Ratio in some of his music and that here the experts don't agree on whether some Golden Ratio patterns that can be discerned are intended or spurious.

"Amen break"



The **"Amen break"** was a brief drum solo performed in 1969 by Gregory Cylvester "G. C." Coleman in the song "Amen, Brother" performed by the 1960s funk and soul outfit The Winstons.

Mathematician Michael S. Schneider saw a wave form of the well-known drum sequence known as the Amen Break. It's a drum 5.2 second sequence performed by Gregory Cylvester Coleman of The Winstons and has been sampled and used by countless artists since it was recorded in the 60s. Schneider, seeing the waveform through the eyes of a math professor, recognized a pattern, a relationship called the Golden Ratio. So he began to analyze the drum sequence and its deeper meaning.

He examined the wave image in his computer and found that Golden Ratio relationships were indicated among the different peaks. He said that *'To appreciate this relationship between the Golden Ratio and sound, it's worthwhile to consider some of the ideal, eternal, unchanging principles of Golden relationships which can only be approximated in nature, and by artists, architects and musicians'.*

It gained fame from the 1980s onwards when four bars (5.2 seconds) sampled from the drum-solo (or imitations thereof) became very widely used as sampled drum loops in hip hop, jungle, break core and drum and bass music.

The song itself achieved fame within the hip hop and subsequent electronic music communities when a former Downstairs Records employee known as Breakbeat Lenny compiled it onto his 1986 *Ultimate Breaks and Beats* bootleg series for DJs.

It created a jarring difference in tempo in the center of the song; it allowed Hip-Hop DJs to extend the beat by switching between two copies of the record on two separate turntables at a danceable tempo while ignoring the rest of the song. The Amen break gain a massive amount of fame in the late 80s hip-hop community, crossing over to the U.K. and European dance music scenes shortly afterwards. Eventually, the song was reissued in its original form at a higher quality sound.